AP Calc BC Convergence Tests

Name: Block: Seat:

n^{th} Term Divergence Test

$$\sum_{n=k}^{\infty} a_n$$

- diverges if $\lim_{n\to\infty} a_n \neq 0$
- diverges if $\lim_{n\to\infty} a_n$ does not exist

1. Determine the convergence of

$$\sum_{n=1}^{\infty} \frac{n-1}{n+1}$$

Geometric Series

The series

$$\sum_{n=0}^{\infty} ar^n$$

- converges only if $-1 < r < 1$
- the sum is $\dfrac{a}{1-r}$

1. Determine the convergence of

$$\sum_{n=1}^{\infty} \left(\frac{-3}{5}\right)^n$$

if it converges, find the sum

p **Series Test**
The series

$$\sum_{n=k}^{\infty} \frac{1}{n^p}$$

- diverges if $p \leq 1$
- converges if $p > 1$

1. Determine the convergence of

$$\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}$$

Direct Comparison Test This test is used when a known series is bigger than the given series.

- If a_n has no negative terms, and a ceiling function $\sum_k^\infty b_n$ converges, then $\sum_k^\infty a_n$ must also converge.
- If a_n has no negative terms, and a floor function $\sum_k^\infty b_n$ diverges, then $\sum_k^\infty a_n$ must also diverge.

1. Use the direct Comparison Test to determine the convergence of

$$\sum_{n=1}^{\infty} \frac{1}{n^2+3}$$

2. Use the direct Comparison Test to determine the convergence of

$$\sum_{n=1}^{\infty} \frac{1}{3^n + 2}$$

3. Use the direct Comparison Test to determine the convergence of

$$\sum_{n=1}^{\infty} \frac{1}{n^{1/2} + 2}$$

Limit Comparison Test This is one of the most useful tests for determining convergence. Suppose $a_n > 0$ and $b_n > 0$ for all $n > N$ where N is a positive integer.

- If $\lim_{n\to\infty} \dfrac{a_n}{b_n} = c$, $0 < c < \infty$, then $\sum a_n$ and $\sum b_n$ behave the same.
- If $\lim_{n\to\infty} \dfrac{a_n}{b_n} = 0$, and b_n converges, then $\sum a_n$ converges.
- If $\lim_{n\to\infty} \dfrac{a_n}{b_n} = \infty$, and b_n diverges, then $\sum a_n$ diverges.

1. Use the Limit Comparison Test to determine the convergence of

$$\sum_{n=1}^{\infty} \frac{3n+2}{(n+1)^2}$$

2. Use the Limit Comparison Test to determine the convergence of

$$\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$$

3. Use the Limit Comparison Test to determine the convergence of

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^3 - 2n}$$

4. Use the Limit Comparison Test to determine the convergence of

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$$

Alternating Series Test for Convergence The series

$$\sum_{n=1}^{\infty}(-1)^{n+1}a_n = a_1 - a_2 + a_3 - a_4 + \dots$$

will converge if all three true:

- Each Term is positive
- The terms are eventually decreasing
- $\lim_{n\to\infty} a_n = 0$

1. Use the AST to determine to determine the convergence of

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$$

2. Use the AST to determine to determine the convergence of

$$\sum_{n=1}^{\infty}\frac{3(-1)^n}{n^2+1}$$

3. Use the AST to determine to determine the convergence of
$$\sum_{n=1}^{\infty} \frac{10(-1)^n}{n!}$$

4. Use the AST to determine to determine the convergence of
$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{e^n - 1}$$

Root Test The next test for convergence or divergence of series works especially well for series involving nth powers.

Let $\sum_{n=k}^{\infty} a_n$ be a series.

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$ exists, then

- If $L < 1$, then the series converges ☺
- If $L > 1$ then the series diverges ☺
- If $L = 1$ then the test is inconclusive ☹

1. Use the Root test to determine the convergence of
$$\sum_{n=1}^{\infty} \left[\frac{n^2+1}{2n^2+1}\right]^n$$

2. Use the Root test to determine the convergence of
$$\sum_{n=1}^{\infty} 3^n e^{-n}$$

3. Use the Root test to determine the convergence of
$$\sum_{n=1}^{\infty} \frac{e^{3n}}{n^n}$$

4. Use the Root test to determine the convergence of
$$\sum_{n=1}^{\infty} \left[\frac{-1}{\arctan n}\right]^n$$

Ratio Test This test is used most often on the AP Exam to determine convergence of a power series.

Let $\sum_{n=k}^{\infty} a_n$ be a series with positive terms and

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$$

- If $L < 1$, then the series converges ☺
- If $L > 1$ then the series diverges ☺
- If $L = 1$ then the test is inconclusive ☹

1. Use the Ratio Test to determine if the following converges:

$$\sum_{n=0}^{\infty} \frac{2^n}{3^n + 1}$$

2. Use the Ratio Test to determine the interval and radius of convergence (the values of x that will make this converge):

$$\sum_{n=0}^{\infty} \frac{nx^n}{10^n}$$

3. Use the Ratio Test to determine the interval and radius of convergence (the values of x that will make this converge):

$$\sum_{n=0}^{\infty}(x+5)^n$$

After the **Ratio Test** find the values of x that will make the limit less than 1. This interval of values of x that make it converge is the **interval of convergence**. Be sure to check the end points of the interval

1. Find the interval of convergence for

$$\sum_{n=4}^{\infty}\frac{nx^n}{4^n(n^2+1)}$$

2. Find the interval of convergence for

$$\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{x^{2n}}{2n}\right)$$

3. Find the interval of convergence for

$$\sum_{n=4}^{\infty} \frac{(x-2)^n}{3n}$$

4. Find the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\ 3^n}$$